Distributed Piezoelectric Segment Method for Vibration Control of Smart Beams

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Introduction

In recent years, smart structures for vibration control of space flexible structures have attracted much attention in the field of vibration control. There have been many significant works on the modeling, basic equations, control laws, finite-element analysis methods, and experiments for smart structures.[1–9] Modal control methods, often used in vibration control, need distributed sensors/actuators to sense/actuate the desired modes. Lee et al.[8,9] presented a modal sensor/actuator designing method by means of reshaping the surface electric pattern of the piezoelectric layers and achieved critical damping of the first mode of a one-dimensional plate. This method, however, is very sensitive to change of structure. Therefore, we use the distributed piezoelectric segment method for modal control of beams. This method includes new means for designing a piezoelectric modal sensor and modal actuator as well as a modal control scheme.

Consider a slender elastic beam. Two piezoelectric laminae, for instance, polyvinylidenefluoride (PVDF), are bonded onto the upper face of the beam as actuator and onto the lower face as sensor. For the sensor lamina, host beam, and actuator lamina, the Young's moduli are Y_1, Y_2, and Y_3; the mass densities are ρ_1, ρ_2, and ρ_3; and the widths are b_1, b_2, and b_3, respectively. The length of the beam is l. The coordinates of the lower and upper surfaces of the sensor lamina are z_0 and z_1, respectively; those of the actuator lamina are z_2 and z_3, respectively. The piezoelectric stress constant is e_{31}^s for the sensor lamina and e_{31}^a for the actuator lamina.

Modal Sensor Design

The charge of the sensor lamina can be derived as

$$q = -e_{31}^s b_1 \int_0^l r_s w''(x, t)\,dx \tag{1}$$

where $r_s = (z_0 + z_1)/2$ is the value of coordinate z at the midplane of the sensor lamina. Differentiating Eq. (1) with respect to t, we get the output electric current of the sensor lamina.

To draw the modal coordinates and modal velocities from the voltage output of the sensor lamina, we develop a strategy of fully distributed piezoelectric segments. The sensor lamina is separated into m_s sensor segments, for example, $S_1, S_2, \ldots, S_{m_s}$. From Eq. (1), the charge of the ith sensor segment caused by the strain of the beam is that

$$q_i(t) = b_1 e_{31}^s \int_{s_i} \varepsilon(x, t)\,dx, \qquad i = 1, 2, \ldots, m_s \tag{2}$$

where $\varepsilon(x, t) = -r_s w''(x, t)$ is the strain of the sensor lamina. Making mode truncation, the transverse displacement of the beam can be expressed as a linear superposition of the former m_s modes of the beam:

$$w(x, t) = \sum_{j=1}^{m_s} \eta_j^*(t) W_j(x) \tag{3}$$

where $W_j(x)$ is the jth mode shape of the beam, and $\eta_j^*(t)$ is the observed value of the jth modal coordinate. Substituting Eq. (3) into Eq. (2) yields

$$\{q\} = e_{31}^s b_1 [\bar{\mathcal{E}}]\{\eta^*\} \tag{4}$$

where $\{q\} = (q_1, q_2, \ldots, q_{m_s})^T$, $\{\eta^*\} = (\eta_1^*, \eta_2^*, \ldots, \eta_{m_s}^*)^T$, and $[\bar{\mathcal{E}}]$ is a $m_s \times m_s$ matrix concerning the integration of strain modes; its elements are

$$\bar{\mathcal{E}}_{ij} = \int_{s_i} -r_s W_j''(x)\,dx, \quad (i = 1, 2, \ldots, m_s; \quad j = 1, 2, \ldots, m_s)$$

The matrix $[\bar{\mathcal{E}}]$ can be made nonsingular by superposing the output charge of segments. Without loss of generality, the matrix $[\bar{\mathcal{E}}]$ is assumed to be nonsingular; the m_s modal coordinates can be solved from Eq. (4):

$$\{\eta^*(t)\} = \frac{1}{b_1 e_{31}^s}[\bar{\mathcal{E}}]^{-1}\{q(t)\} \tag{5}$$

and, similarly

$$\{\dot{\eta}^*(t)\} = \frac{1}{b_1 e_{31}^s}[\bar{\mathcal{E}}]^{-1}\{I(t)\} \tag{6}$$

where $\{I(t)\}$ is the vector that consists of the output currents of m_s sensor segments.

Equations (5) and (6) show the process of mode observation performed by modal sensors; the observed values of former m_s modal coordinates and modal velocities can be obtained from the output charge and current of all sensor segments.

Modal Actuator Design

The piezoelectric modal actuator is designed by adjusting the distribution in space of the voltage of the actuator lamina. The differential equation of motion of the smart beam can be derived as

$$\rho A(x)\ddot{w} + (YJ(x)w'')'' = -b_3 e_{31}^a (r_a V)'' \tag{7}$$

where

$$YJ(x) = \sum_{k=1}^{3} b_k Y_k \left(z_k^3 - z_{k-1}^3\right) \Big/ 3$$

is the bending stiffness of the beam, $\rho A(x)$ is the mass density in a unit length of the beam, V is the voltage exerted on the actuator lamina in direction z, and r_a is the z coordinate of the midplane of the actuator lamina. Equation (7) indicates that the motion of the smart beam responds to the voltage applied to the actuator lamina; it is also called the actuator equation.

Expressing displacement $w(x, t)$ as a superposition of modes $W_j(x)$ and employing the orthogonality of modes, we obtain the modal equations of motion

$$\ddot{\eta}_j(t) + \omega_j^2 \eta_j(t) = -b_3 e_{31}^a \int_0^l (r_a V)'' W_j(x)\,dx, \qquad j = 1, 2, \ldots \tag{8}$$

It can be seen that only some specified modes can be actuated by the voltage applied to the piezoelectric actuator lamina through modulating the distribution, in space, of the voltage. To this end, we design the voltage to be determined by

$$V(x, t) = \frac{1}{r_a}\sum_{j=1}^{n} p_j(t)\,\mathcal{M}_j(x) \tag{9}$$

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where n is the number of the order of required controlled modes, $n \le m_s$, and $\mathcal{M}_j(x) = -YJ(x)W_j''(x)$ is jth modal bending moment. Substituting Eq. (9) into Eq. (8) by using the orthogonality of modes yields

$$\begin{aligned} &\ddot{\eta}_k(t) + \omega_k^2\eta_k(t) = b_3 e_{31}^a \omega_k^2 p_k(t), \qquad k = 1, 2, \ldots, n \\ &\ddot{\eta}_k(t) + \omega_k^2\eta_k(t) = 0, \qquad k = n + 1, \ldots \end{aligned} \tag{10}$$

Equation (10) shows that the former n modes can be controlled.

To realize the distribution, in space, of the voltage in Eq. (9), the piezoelectric actuator lamina is divided into m_a segments, $A_1, A_2, \ldots, A_{m_a}$, $m_a > n$, the length of the ith actuator segment is ΔX_i. By applying constant voltage to each segment, the continuous distribution, in space, of voltage $V(x, t)$ is approximated by "piecewise constant voltage." The applied voltage to the actuator segments is

$$\{V(t)\} = [C][\bar{\mathcal{M}}]\{P(t)\} \tag{11}$$

where

$$\{V(t)\} = \{V_1(t), V_2(t), \ldots, V_{m_a}(t)\}^T, \qquad [C] = \mathrm{diag}(1/r_{ai}\Delta X_i)$$

$$P(t) = [p_1(t), p_2(t), \ldots, p_n(t)]^T$$

represents the modal control force vector, r_{ai} is the z coordinate of the midplane of the ith actuator segment, and the elements of matrix $[\bar{\mathcal{M}}]$ are

$$\bar{\mathcal{M}}_{ij} = \int_{A_i} \mathcal{M}_j(x)\,\mathrm{d}x, \qquad (i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n)$$

In this way, the required voltage on each piezoelectric actuator segment can be determined from Eq. (11), and the modal control can be approximately realized by applying these voltages to actuator segments.

Modal Control

The modal control of the beam considered can be realized by designing the modal control forces $p_k(t)$ in the modal equation of motion (10). By using negative feedback for modal coordinates and velocities, the $p_k(t)$ are designated as

$$\{P(t)\} = -[G]\{\eta^*(t)\} - [H]\{\dot{\eta}^*(t)\} \tag{12}$$

where $[G] = \mathrm{diag}(g_1, g_2, \ldots, g_n)$ and $[H] = \mathrm{diag}(h_1, h_2, \ldots, h_n)$ are diagonal modal gain matrices. Inserting Eq. (12) into Eq. (10) gives the closed-loop equations

$$\ddot{\eta}_k(t) + \omega_k^2\eta_k(t) = -b_3 e_{31}^a \omega_k^2 g_k \eta_k^*(t) - b_3 e_{31}^a \omega_k^2 h_k \dot{\eta}_k^*(t), \qquad k = 1, 2, \ldots, n \tag{13}$$

Substituting Eq. (12) into Eq. (11) yields voltage distribution, and applying the m_a voltages obtained on the actuator segments, respectively, can realize the modal control of the beam.

Simulation and Conclusion

As an example, consider a uniform cantilever beam to which two PVDF laminae are bonded as sensor and actuator: $l = 400$ mm, $Y_2 = 210$ MPa, $\rho_2 = 8000\,\mathrm{kg/m^3}$, thickness $H_2 = 1$ mm, $Y_1 = Y_3 = 2$ MPa, $\rho_1 = \rho_3 = 1780\,\mathrm{kg/m^3}$, $H_1 = H_3 = 0.3$ mm, $b_1 = b_2 = b_3 = 30$ mm, $e_{31}^s = e_{31}^a = 6 \times 10^{-2}$ N/Vm. To control the beam's vibration caused by an initial impulse of 5×10^{-4} Ns on its midpoint, both sensor and actuator lamina are separated into seven equal segments. We control the first four modes and the sixth mode of the beam and choose $h_1 = 0.9$, $h_2 = 0.15$, $h_3 = 0.05$, $h_4 = 0.03$, $h_6 = 0.01$. In this case, the equivalent damping ratios of five modes are $\zeta_1 = 0.0249$, $\zeta_2 = 0.0248$, $\zeta_3 = 0.0213$, $\zeta_4 = 0.0222$, and $\zeta_6 = 0.0139$, respectively. To remove the observation spillover, the low-pass filter is used. The displacement response of the free end is shown in Fig. 1, and the voltage distributions for the seven actuator segments are shown in Fig. 2. The vibration of the beam is effectively suppressed within 7 s, and the maximum voltage is less than 280 V.

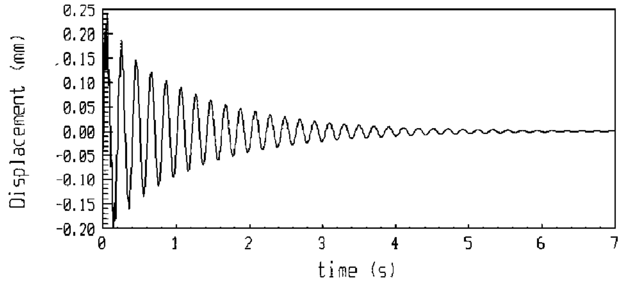

Fig. 1 Displacement of the free end.

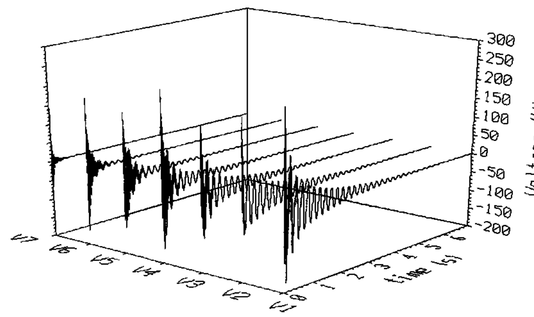

Fig. 2 Voltage distribution of the actuator segments.

There are three advantages to the method presented in this Note: 1) changing the order of controlled modes can be recognized by software, but it is not necessary to change the shape of the piezoelectric sensor and actuator laminae on the structures; 2) it may not be necessary to change the sensor and actuator laminae for a change in the basic structure; and 3) the sensor/actuator can sense/actuate not only the local strain or strain rate but also the entire strain or strain rate. Therefore, this method is suitable for vibration control with an onboard computer.

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